Comparing Planar and Spherical Geometry

Complete the table below to compare and contrast lines in the system of plane Euclidean geometry and lines (great circles) in spherical geometry.

<table>
<thead>
<tr>
<th></th>
<th>On the plane</th>
<th>On the sphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Is the length of a line finite or infinite?</td>
<td>infinite</td>
<td>finite</td>
</tr>
<tr>
<td>2. Describe the shortest path that connects two points.</td>
<td>line segment</td>
<td>arc of a great circle</td>
</tr>
<tr>
<td>3. Can you extend a line forever?</td>
<td>yes</td>
<td>no (line meets itself)</td>
</tr>
<tr>
<td>4. How many parts (and are they finite or infinite) will two points divide a line?</td>
<td>1 finite, 2 infinite</td>
<td>2 finite</td>
</tr>
<tr>
<td>5. How many lines pass through any two different points?</td>
<td>exactly one</td>
<td>2 non-polar: one per pt, 2 polar: infinite</td>
</tr>
<tr>
<td>6. How many lines are parallel to a given line and pass through a given point not on the given line?</td>
<td>exactly one</td>
<td>none (parallel lines do not exist in spherical geom.)</td>
</tr>
<tr>
<td>7. If three points are collinear, exactly one is between the other two. (True or false)</td>
<td>true</td>
<td>false</td>
</tr>
</tbody>
</table>

For each property listed from plane Euclidean geometry, write a corresponding statement for spherical geometry.

8. Two distinct lines with no point of intersection are parallel.
   Two distinct lines with no point of intersection do not exist

9. Two distinct intersecting lines intersect in exactly one point.
   Two distinct intersecting lines intersect in exactly two points

10. A pair of perpendicular lines divides the plane into four infinite regions.
    A pair of ⊥ lines divides the sphere into 4 finite regions

11. A pair of perpendicular lines intersects once and creates four right angles.
    A pair of ⊥ lines intersects twice and creates 8 right angles

12. Parallel lines have infinitely many common perpendicular lines.
    Parallel lines do not exist.

13. There is only one distance that can be measured between two points.
    There are two distances that can be measured between a points

14. There is exactly one line passing through two points.
    There is exactly one line passing through 2 nonpolar points.
Choose one of the following answers for each question:

A) true on a plane
B) true on a sphere
C) true on both a plane and a sphere
D) true on neither a plane or a sphere

15. A line is an infinite set of points. C
16. A line is continuous (no "holes" or gaps). C
17. Through any two points, there is exactly one line. A
18. There exists at least one pair of points through which more than one line can be drawn. B
19. A polygon may have two sides. A
20. Each angle of an equilateral triangle must be 60°. A
21. Each angle of an equilateral triangle may be 45°. D
22. Each angle of an equilateral triangle may be 120°. B
23. A line is bounded. (that is, it can fit into a closed box) B
24. There is no greatest distance between two points. A
25. Two lines can share no points. A
26. Two distinct lines can share two points. B
27. Two distinct lines can share more than two points. D
28. The sum of the angles of a triangle is always the same number. A
29. A triangle can have at most one right angle. A
30. Three lines may be perpendicular to each other (that is, line a \( \perp \) line b \( \perp \) line c \( \perp \) line a) A
31. Three lines may be parallel to each other. A
32. Three lines may intersect in three points. (each of the lines intersects the other two lines) A
33. Three lines may intersect in two points. (each of the lines intersects the other two lines) B
34. Three lines may intersect in four points. D
35. Vertical angles are congruent. C